SADLER UNIT 3 CHAPTER 6

EXERCISE 6A

Q1. $x = 3$
$y = -2$
$z = 5$

Q2 $x = 4$
$y = 7$
$z = -2$

Q3. $z = 1$
$2y + 5 = 15$
$2y = 10$
$y = 5$
$2x + 5 + 1 = 4$
$2x = -2$
$\underline{\underline{x = -1}}$

Q4. $-z = 3$
$z = -3$
$3y + 2(-3) = 6$
$3y = 12$
$y = 4$
$x - 2(-3) = 7$
$\underline{\underline{x = 1}}$

Q5 $2z = 12$
$z = 6$
$5y - 3(6) = 2$
$5y = 20$
$y = 4$
$x + 3(4) + 2(6) = 27$
$x + 24 = 27$
$\underline{\underline{x = 3}}$

Q6. $-3z = 9$
$z = -3$
$3y - 2(-3) = 0$
$3y = -6$
$y = -2$
$2x + (-2) + (-3) = -3$
$2x = 2$
$\underline{\underline{x = 1}}$

Q7. $\left[\begin{array}{cc|c} 3 & 2 & 10 \\ 1 & -4 & 8 \end{array}\right]$

Q8 $\left[\begin{array}{cc|c} -1 & 5 & 12 \\ 2 & 3 & 2 \end{array}\right]$

Q9 $\left[\begin{array}{ccc|c} 1 & 4 & 3 & 18 \\ 3 & 1 & 2 & 11 \\ 5 & 2 & 1 & 12 \end{array}\right]$

Q10. $\left[\begin{array}{ccc|c} 2 & 0 & 3 & 14 \\ 4 & 1 & -1 & 0 \\ 2 & 1 & 6 & 26 \end{array}\right]$

Q11. $\left[\begin{array}{ccc|c} 3 & 2 & 0 & 8 \\ 1 & 0 & 2 & 8 \\ 0 & 2 & -1 & -1 \end{array}\right]$

Q12 $\left[\begin{array}{ccc|c} 1 & 3 & -5 & 2 \\ 2 & 1 & 7 & 37 \\ -1 & 0 & 1 & 3 \end{array}\right]$

Q13. $x + 3y = 34$ ① $\times 2$
$2x + 5y = 59$ ②
$-\ 2x + 6y = 68$ ③
$-y = -9$
$y = 9$
Sub into ①.
$\therefore x = 34 - 3(9)$
$\underline{\underline{x = 7}}$

Q14. $2x + 3y = 4$ ① $\times 2$
$4x + 9y = 2$ ②
$-\ 4x + 6y = 8$ ③
$3y = -6$
$\underline{\underline{y = -2}}$
Sub into ①
$2x = 4 - 3(-2)$
$2x = 10$
$x = 5$ //

Q15 $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & 7 \\ 3 & 3 & 1 & 14 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$

$R_3 \Rightarrow R_3 - 3R_1$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & 7 \\ 0 & -3 & -2 & -7 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$

$R_3 \Rightarrow R_3 + 3R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 7 & 14 \end{array}\right]$

$7z = 14$
$\underline{\underline{z = 2}}$

$y + 3(2) = 7$
$\underline{\underline{y = 1}}$

$x + 2(1) + 2 = 7$
$\underline{\underline{x = 3}}$

Q16.

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 4 & 6 \\ 2 & 3 & -3 & 20 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 - 2R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & -5 & -8 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & -8 & 8 \end{array}\right]$

$-8z = 8$
$\underline{\underline{z = -1}}$

$y + 3(-1) = 0$
$\underline{\underline{y = 3}}$

$x + 3 - 1 = 6$
$\underline{\underline{x = 4}}$

Q17.

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 2 & 1 & 3 & 8 \\ 5 & 1 & 0 & 35 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 0 & 1 & -5 & 10 \\ 0 & 1 & -20 & 40 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 0 & 1 & -5 & 10 \\ 0 & 0 & -15 & 30 \end{array}\right]$$

$-15z = 30$

$z = -2$

$y - 5(-2) = 10$

$y + 10 = 10$

$y = 0$

$x + 4(-2) = -1$

$x = 7$

Q18

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 3 & 2 & -1 \\ 3 & 7 & -2 & 6 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 4 & -7 \\ 0 & 1 & 1 & -3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 4 & -7 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

$5z = -10$

$z = -2$

$-y + 4(-2) = -7$

$-y = 1$

$y = -1$

$x - 2 + 2 = 3$

$x = 3$

Q19.

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 11 \\ 1 & 2 & -1 & 15 \\ 3 & 9 & 1 & 16 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 \Leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 15 \\ 2 & 1 & 0 & 11 \\ 3 & 9 & 1 & 16 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 15 \\ 0 & -3 & 2 & -19 \\ 0 & 3 & 4 & -29 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 15 \\ 0 & -3 & 2 & -19 \\ 0 & 0 & 6 & -48 \end{array}\right]$$

$6z = -48$

$z = -8$

$-3y + 2(-8) = -19$

$-3y = -3$

$y = 1$

$x + 2(1) - (-8) = 15$

$x + 10 = 15$

$x = 5$

Q20.

$$\left[\begin{array}{ccc|c} 2 & 4 & -3 & 1 \\ 2 & 5 & -2 & 5 \\ 3 & 7 & -3 & 7 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow 2R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -3 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 2 & 3 & 11 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -3 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z = 3$

$y + 3 = 4$

$y = 1$

$2x + 4 - 9 = 1$

$2x = 6 \Rightarrow x = 3$

Q21. $\left[\begin{array}{ccc|c} 3 & 4 & 5 & 14 \\ 5 & 7 & 6 & 13 \\ 1 & 1 & 1 & 3 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$

$R_1 \Leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 5 & 7 & 6 & 13 \\ 3 & 4 & 5 & 14 \end{array}\right] \begin{matrix} R_1 \\ R_2 \Rightarrow R_2 - 5R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 2 & 1 & -2 \\ 0 & 1 & 2 & 5 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \Rightarrow 2R_3 - R_2 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 3 & 12 \end{array}\right]$

$3z = 12$

$z = 4$

$2y + 4 = -2$

$2y = -6$

$y = -3$

$x - 3 + 4 = 3$

$x = 2$

Q22. $\left[\begin{array}{ccc|c} 2 & 0 & 1 & 4 \\ 2 & 3 & 3 & 3 \\ 5 & 1 & 3 & 10 \end{array}\right] \begin{matrix} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow 2R_3 - 5R_1 \end{matrix}$

$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 4 \\ 0 & 3 & 2 & -1 \\ 0 & 2 & 1 & 0 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \Rightarrow 3R_3 - 2R_2 \end{matrix}$

$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 4 \\ 0 & 3 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{array}\right]$

$-z = 2$

$z = -2$

$3y - 4 = -1$

$3y = 3$

$y = 1$

$2x - 2 = 4$

$x = 3$ //

Q23. $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 3 & 2 & 1 & 7 \\ 5 & 4 & 4 & 19 \end{array}\right] \begin{matrix} R_1 \\ R_2 \Rightarrow R_2 - 3R_1 \\ R_3 \Rightarrow R_3 - 5R_1 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & -1 & -5 & -11 \\ 0 & -1 & -6 & -11 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & -1 & -5 & -11 \\ 0 & 0 & -1 & 0 \end{array}\right]$

$-z = 0$

$z = 0$

$-y + 0 = -11$

$y = 11$

$x + 11 + 0 = 6$

$x = -5$

Q24. $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & -3 & -2 \\ 1 & 2 & 2 & 1 & 0 \\ 2 & 3 & 2 & 7 & 4 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_1 \\ R_4 \Rightarrow R_4 - 2R_1 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & -3 & -2 \\ 0 & 1 & 3 & -2 & 1 \\ 0 & 1 & 4 & 1 & 6 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \\ R_4 \Rightarrow R_4 - R_2 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & -3 & -2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 4 & 8 \end{array}\right] \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \Rightarrow R_4 - 2R_3 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & -3 & -2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right]$

$2z = 2$ $\quad y + 1 = 3$ $\quad x + 4 - 3 = -2$

$z = 1$ $\quad y = 2$ $\quad x = -3$

$\therefore w + (-3) - 2 + 3 = -1$

$w = 1$

Q25

$5x + 3y = 270$ ①

$2x + 4y = 220$ ② $(\div 2)$

$x + 2y = 110$ ③

$x = 110 - 2y$

$5(110-2y) + 3y = 270$

$550 - 10y + 3y = 270$

$-7y = -280$

$y = 40$

$x = 110 - 80$

$x = 30$

∴ 30 type A containers and 40 type B containers.

Q26

$250p + 500q + 200r = 8000$

$5p + 10q + 4r = 160$ ①

$10p + 5q + 20r = 470$

$2p + q + 4r = 94$ ②

$50p + 100q + 100r = 2800$

$p + 2q + 2r = 56$ ③

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 56 \\ 2 & 1 & 4 & 94 \\ 5 & 10 & 4 & 160 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 56 \\ 0 & -3 & 0 & -18 \\ 0 & 0 & -6 & -120 \end{array}\right]$$

$-6r = -120$

$r = 20$

$-3q = -18$

$q = 6$

$p + 2(6) + 2(20) = 56$

$p + 52 = 56$

$p = 4$

∴ 4 P tablets, 6 Q tablets and 20 R tablets.

Q27.

$0.5x + 0.3y + 0.8z = 610$

$5x + 3y + 8z = 6100$ ①

$0.1x + 0.5y + 0.1z = 180$

$x + 5y + z = 1800$ ②

$0.4x + 0.2y + 0.1z = 210$

$4x + 2y + z = 2100$ ③

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 1800 \\ 4 & 2 & 1 & 2100 \\ 5 & 3 & 8 & 6100 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 4R_1 \\ R_3 \Rightarrow R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 1800 \\ 0 & -18 & -3 & -5100 \\ 0 & -22 & 3 & -2900 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 1800 \\ 0 & -18 & -3 & -5100 \\ 0 & -40 & 0 & -8000 \end{array}\right]$$

$-40y = -8000$

$y = 200$

$-18(200) - 3z = -5100$

$3z = 5100 - 3600$

$3z = 1500$

$z = 500$

$x + 5(200) + 200 = 1800$

$x + 1200 = 1800$

$x = 600$

∴ 600kg of A, 500kg of B and 200kg of C.

EXERCISE 6B

Q1 $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & k & 5 \end{array}\right]$

$k = 0$

Q2. $\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & k-2 & 3 \end{array}\right]$

$k - 2 = 0$

$k = 2$

Q3. $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 2k+1 & 2 \end{array}\right]$

$2k + 1 = 0$

$k = -\frac{1}{2}$

Q4. $\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & k & 2 \\ 0 & -1 & 3 & 5 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & k & 2 \\ 0 & 0 & 3+k & 7 \end{array}\right]$

$3 + k = 0$

$k = -3$

Q5 $\left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 0 & 3 & k & 4 \\ 0 & 2 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow 3R_3 - 2R_2 \end{array}$

$\left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 0 & 3 & k & 4 \\ 0 & 0 & 3-2k & 1 \end{array}\right]$

$3 - 2k = 0$

$2k = 3$

$k = \frac{3}{2}$

Q6 $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & k \\ 0 & -2 & 6 & -4 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + 2R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & k \\ 0 & 0 & 0 & -4+2k \end{array}\right]$

$k \in \mathbb{R}, k \neq 2.$

Q7. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & k-2 & 4 \end{array}\right]$

$k - 2 = 0$

$k = 2$

Q8 $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & k+1 & 5 \end{array}\right]$

$k + 1 = 0$

$k = -1$

Q9. $\left[\begin{array}{ccc|c} 1 & 2 & k & 1 \\ 2 & -3 & 1 & 5 \\ 3 & -1 & 4 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & k & 1 \\ 0 & -7 & 1-2k & 3 \\ 0 & -7 & 4-3k & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & k & 1 \\ 0 & -7 & 1-2k & 3 \\ 0 & 0 & 3-k & -3 \end{array}\right]$

$3 - k = 0$

$k = 3$

Q10 $\left[\begin{array}{ccc|c} 1 & 3 & -6 & 3 \\ 1 & 1 & 1 & 0 \\ 3 & 5 & k+1 & 2 \end{array}\right]$ R_1 ; $R_2 \Rightarrow R_2 - R_1$; $R_3 \Rightarrow R_3 - 3R_1$

$\left[\begin{array}{ccc|c} 1 & 3 & -6 & 3 \\ 0 & -2 & 7 & -3 \\ 0 & -4 & k+19 & -7 \end{array}\right]$ R_1 ; R_2 ; $R_3 \Rightarrow R_3 - 2R_2$

$\left[\begin{array}{ccc|c} 1 & 3 & -6 & 3 \\ 0 & -2 & 7 & -3 \\ 0 & 0 & k+5 & -1 \end{array}\right]$

$k + 5 = 0$

$k = -5$

Q11 $\left[\begin{array}{ccc|c} 1 & 4 & 2 & -7 \\ 2 & 1 & k & -1 \\ 3 & -2 & 4 & 1 \end{array}\right]$ R_1 ; $R_2 \Rightarrow R_2 - 2R_1$; $R_3 \Rightarrow R_3 - 3R_1$

$\left[\begin{array}{ccc|c} 1 & 4 & 2 & -7 \\ 0 & -7 & k-4 & 13 \\ 0 & -14 & -2 & 22 \end{array}\right]$ R_1 ; R_2 ; $R_3 \Rightarrow R_3 - 2R_2$

$\left[\begin{array}{ccc|c} 1 & 4 & 2 & -7 \\ 0 & -7 & k-4 & 13 \\ 0 & 0 & 6-2k & -4 \end{array}\right]$

$6 - 2k = 0$

$2k = 6$

$k = 3$

Q12 $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 4 \\ -1 & 5 & k+1 & 6 \\ 2 & -1 & 1 & 5 \end{array}\right]$ R_1 ; $R_2 \Rightarrow R_2 + R_1$; $R_3 \Rightarrow R_3 - 2R_1$

$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 4 \\ 0 & 6 & k+4 & 10 \\ 0 & -3 & -5 & -3 \end{array}\right]$ R_1 ; R_2 ; $R_3 \Rightarrow 2R_3 + R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 4 \\ 0 & 6 & k+4 & 10 \\ 0 & 0 & -6+k & 4 \end{array}\right]$

$-6 + k = 0$

$k = 6$

Q13 $\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & k & 0 \end{array}\right]$

$k = 0$

Q14 $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2k+1 & 0 \end{array}\right]$

$2k + 1 = 0$

$k = -\frac{1}{2}$

Q15 $\left[\begin{array}{ccc|c} 1 & -3 & 5 & 5 \\ 0 & -1 & 2 & 8 \\ 0 & 0 & k^2-4 & k+2 \end{array}\right]$

$k^2 - 4 = 0$ and $k + 2 = 0$

$k = -2$

Q16. $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 5 \\ 0 & 3 & 2 & 5 \\ 0 & 0 & 2k & 0 \end{array}\right]$

$2k = 0$

$k = 0$

Q17 $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & k & 0 & 0 \end{array}\right]$

$k = 0$

Q18. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 2 & -4 & 6 & -2 \\ -1 & 2 & k & 1 \end{array}\right]$ R_1 ; $R_2 \Rightarrow R_2 - 2R_1$; $R_3 \Rightarrow R_3 + R_2$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3+k & 0 \end{array}\right]$

$k \in \mathbb{R}$, as already infinitely many solutions.

Q19 $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & 3 & k & 2 \\ 4 & 11 & -5 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - 4R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 5 & k-2 & -4 \\ 0 & 15 & -9 & -12 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - 3R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 5 & k-2 & -4 \\ 0 & 0 & -3k+6 & 0 \end{array}\right]$ -9

$-9 - 3k + 6 = 0$

$-3 - 3k = 0$

$3k = -3$

$k = -1$

Q20. $\left[\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 1 & 5 & k-2 & 3 \\ 2 & k+1 & -7 & 9 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 - 2R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 0 & 2 & k & -1 \\ 0 & k-5 & -3 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 0 & 2 & k & -1 \\ 0 & k-3 & k-3 & 0 \end{array}\right]$

$\therefore k - 3 = 0$

$k = 3$

Q21. $\left[\begin{array}{cc|c} 1 & p & 5 \\ 2 & 3 & q \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \end{array}$

$\left[\begin{array}{cc|c} 1 & p & 5 \\ 0 & 3-2p & q-10 \end{array}\right]$

a) $3 - 2p = 0$ and $q - 10 = 0$

$2p = 3$ $\quad q = 10$

$p = \frac{3}{2}$

b) $3 - 2p = 0$ and $q \neq 10$.

$p = \frac{3}{2}$

c) $p \neq \frac{3}{2}$ and $q \neq 10$.

Q22 $\left[\begin{array}{cc|c} p & 4 & 6 \\ 9 & 6 & q \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow 2R_2 - 3R_1 \end{array}$

$\left[\begin{array}{cc|c} p & 4 & 6 \\ 18-3p & 0 & 2q-18 \end{array}\right]$

a) $18 - 3p = 0$ and $2q - 18 = 0$

$3p = 18$ $\quad 2q = 18$

$p = 6$ $\quad q = 9$

b) $p = 6$ and $q \neq 9$

c) $p \neq 6$ and $q \neq 9$

Q23. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 3 & -2 & 7 \\ 3 & 4 & p & q \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 - 3R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & 4 \\ 0 & -2 & p-3 & q-9 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + 2R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & p-9 & q-1 \end{array}\right]$

$p = 9$ and $q = 1$

Q24. $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 2 & 8 & -2 & q \\ 1 & -3 & p & -1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - 2R_1 \\ R_3 \Rightarrow R_3 - R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 2 & 0 & q-4 \\ 0 & -6 & p+1 & -3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + 3R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 2 & 0 & q-4 \\ 0 & 0 & p+1 & 3q-15 \end{array}\right]$

$p + 1 = 0$ and $3q - 15 = 0$

$p = -1$ $\quad q = 5$

Q25 $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 1 & 1 & 1 & 7 \\ -1 & 5 & p & -4 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 + R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 3 & 0 & 9 \\ 0 & 3 & p+1 & -6 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & p+1 & -15 \end{array}\right]$

$p \neq -1$

Q26. $\left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 5 & 2 & -1 & 2 \\ -3 & 1 & p & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow 2R_2 - 5R_1 \\ R_3 \Rightarrow 2R_3 + 3R_1 \end{array}$

$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 5 & 2p & 5 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + 5R_2 \end{array}$

$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 2p-10 & 0 \end{array}\right]$

$2p - 10 \neq 0$

$2p \neq 10$

$p \neq 5$

Q27. $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ -1 & 3 & 1 & 5 \\ 2 & 6 & -2 & 10 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 + R_1 \\ R_3 \Rightarrow R_3 - 2R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 0 & 6 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right]$

∴ Infinitely many solutions as $0x + 0y + 0z = 0$.

Q28 $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -3 & 1 \\ 0 & 2k-1 & 0 & m+1 \end{array}\right]$

a) $2k - 1 \neq 0$

$k \neq \frac{1}{2}$ and $m \in \mathbb{R}$.

b) $2k - 1 = 0$ and $m \neq -1$

$k = \frac{1}{2}$

c) $2k - 1 = 0$ and $m = -1$

$k = \frac{1}{2}$

Q29. $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 12 \\ -1 & -2 & 1 & 3 \\ 8 & 7 & p & q \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 + R_1 \\ R_3 \Rightarrow R_3 - 8R_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 12 \\ 0 & -3 & 3 & 15 \\ 0 & 15 & p-16 & q-96 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 + 5R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 12 \\ 0 & -3 & 3 & 15 \\ 0 & 0 & p-1 & q-21 \end{array}\right]$

a) If $p = 1$, $q = 10$

$\left[\begin{array}{ccc|c} 0 & 0 & 0 & -11 \end{array}\right]$

∴ No solution

b) If $p = 1$, $q = 21$

$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right]$

∴ Infinitely many solutions.

c) If $p = 7$, $q = 45$

$\left[\begin{array}{ccc|c} 0 & 0 & 6 & 24 \end{array}\right]$

$6z = 24$ $\quad -y + 4 = 5$ $\quad x = 3$

$z = 4$ $\quad -y = 1$

$y = -1$

Q30.

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & m \\ 1 & k & -3 & 7 \\ 4 & -1 & -3 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \Rightarrow R_2 - R_1 \\ R_3 \Rightarrow R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & m \\ 0 & k+1 & -3 & 7-m \\ 0 & 3 & -3 & 3-4m \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \Rightarrow R_3 - R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & m \\ 0 & k+1 & -3 & 7-m \\ 0 & 2-k & 0 & -4-3m \end{array}\right]$$

a) $2-k \neq 0$ and $m \in \mathbb{R}$

$k \neq 2$

b) $2-k = 0$ and $-4-3m \neq 0$

$k = 2$ $\quad 3m \neq -4$

$m \neq \frac{-4}{3}$

c) $2-k = 0$ and $-4-3m = 0$

$k = 2$ $\quad m = \frac{-4}{3}$